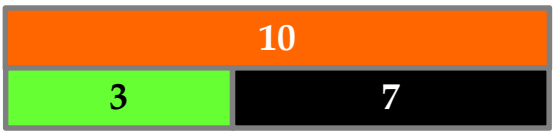
Bar Models Progression - Year 1

Addition and Subtraction

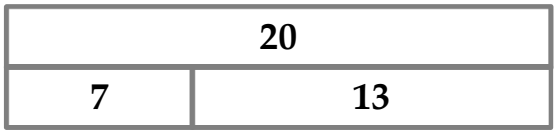
- Represent and use number bonds and related subtraction facts within 20
- Add and subtract one-digit and two-digit numbers to 20, including zero
- Solve one-step problems that involve addition and subtraction

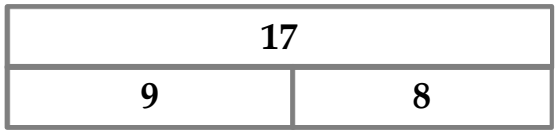
Children use Cuisenaire rods to explore numbers, making number bonds to numbers within 10.
What happens if you change the '7' for an '8'?
Which bar do I need on the top if I exchange the '3' for '2'?



Once confident, they use the same rods to begin exploring subtraction. '7' is 3 less than 10, so 10 – 3 = 7.
Use the rods to show 4 + 5. Can you use the rods to help you fill the gap: 9 – ___ = 4.

When children are confident in using rods to show addition and subtraction facts within 10, they should progress to drawing / completing bar models to show addition and subtraction within 20. (Bars might be drawn for them at this age)
Think about how long each bar should be.
How can you use the bar model for 7 + 3 to complete the model for 7 + 13?





Possible questions:

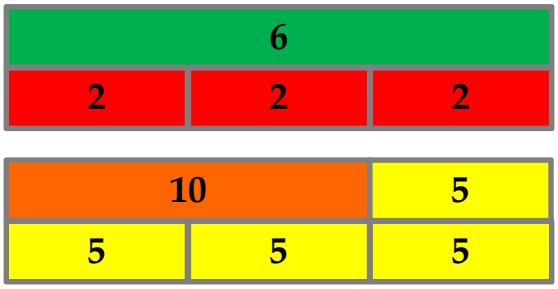
- What is 4 + 5? Show it with a bar model.
- What is 8 – 3? Show it with a bar model.
- Tara had 3 apples and Sara had 4 apples. How many do they have altogether? Show it with a bar model.
- Billy has 4 pencils. Sam has 5 more pencils than Billy. How many does Sam have? Show it with a bar model.
- Amy has 8 sweets and the she gives 5 to her friend. How many does she have left? Show it with a bar model.
- Tom has 7 toy dinosaurs. George has 5 less than Tom. How many does George have? Show it with a bar model.
- Fill in the missing number: 7 + _____ = 16. Show it with a bar model.
- Fill in the missing number: 7 = ____ – 9. Show it with a bar model.

Bar Models Progression - Year 1

Multiplication and Division

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Children begin to use rods to count in 2s, 5s and 10s. Higher attaining children might begin to observe multiplication facts such as 'three 2s are 6'.
If we had another two, what bar would we need on the top?
What could we do if the longest bar wasn't long enough?

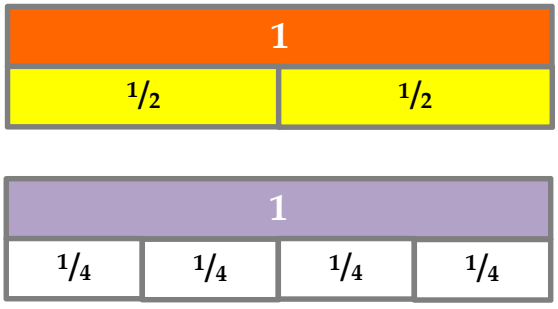


Possible questions:

- Three children each had 2 sweets. How many sweets were there altogether? Show it with a bar model.
- Six children have 5 crayons each. How many crayons are there altogether? Show it with a bar model.

Fractions

- Recognise, find and name a half as one of two equal parts of an object, shape or quantity
- Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity



Children are introduced to the fact that the rods do not need to represent the numbers one to ten. The bars are used to show different fractions.
How many halves did you need to make one whole?
How many quarters did you need to make one whole?

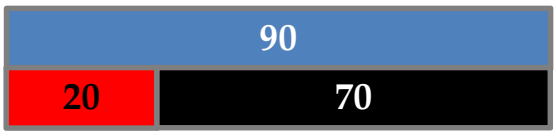
Possible questions:

- What fraction of the purple is each white piece?
- If the value of the orange rod is ten, what is the value of each yellow rod?
- Extension: What if the value of the orange rod is 6? 100? Etc.

Bar Models Progression - Year 2

Addition and Subtraction

- Solve problems with addition and subtraction, using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- Add and subtract numbers using concrete objects, pictorial representations, and mentally
- show that addition of 2 numbers can be done in any order (commutative) and subtraction of 1 number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems



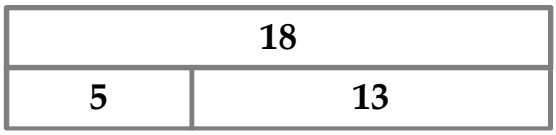
Children continue to use Cuisenaire rods to explore numbers, making number bonds to numbers within 10, and beginning to add three digits together.
What happens if you change the '7' for an '8'?

They continue to explore subtraction using initially two, and then three digits.
Can you use the rods to show 10 – 5 – 3?

Children continue to scale the rods, understanding that they do not need to represent one to ten. They use this to derive addition and subtraction facts up to 100, using their knowledge of number bonds to ten.
If the white bar is worth 10, what is the green bar worth?

Children continue to become familiar with addition and subtraction facts within 20.

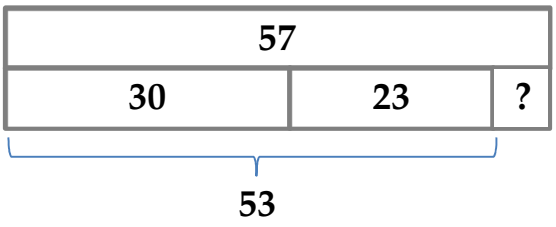
They begin to think more about the relationship between the bars in the model, using each model to show commutivity in addition and explore inverse relationships.
Can you use this bar model to write 4 number sentences?



5 + 13 = 18 13 + 5 = 18
18 – 5 = 13 18 – 13 = 5

Possible questions:

- What is 4 + 3 + 2? Show it with a bar model.
- What is 8 – 3 - 2? Show it with a bar model.
- Micky had 12 marbles and Sarah had 6 marbles. How many did they have altogether?
- Three friends share out 57 playing cards. Sam took 30 cards and Poppy took 23. How many did Andrew have?
- Use a model to show 34 + 62. What other number facts does your model show?



30 + 23 = 53 57 – 53 = 4

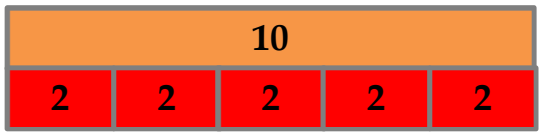
Bar Models Progression - Year 2

Multiplication and Division

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- Show that multiplication of 2 numbers can be done in any order (commutative) and division of 1 number by another cannot
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

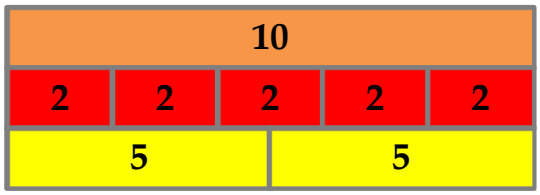
Children use bar models to represent times table facts. Multiplication is linked to repeated addition.

Models help children to build an understanding of their times table facts, by using nearby fact.
This bar model shows the 5 x 2 = 10. What happens if we add one more 2? (we now have 6 x 2 = 12)



2 + 2 + 2 + 2 + 2 = 10

5 x 2 = 10



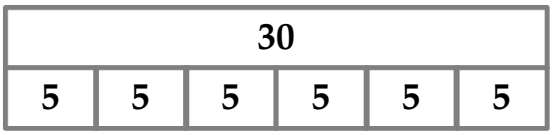
Children use rods to show community in multiplication eg. 5 x 2 = 2 x 5 = 10.

This model can then be used to show 4 number facts:

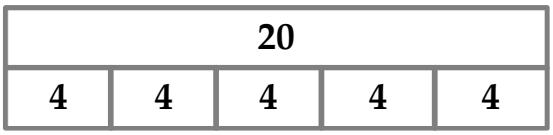
5 x 2 = 10 2 x 5 = 10 10 ÷ 5 = 2 10 ÷ 2 = 5

Simple multiplication and division word problems are introduced and modelled with a bar model.

eg. Six children all bring 5 pencils to school. How many pencils were there altogether?



A teacher split a total of 20 house points equally between 5 children. How many do they get each?



*Reinforce with children the need for numbers on the bottom to add up to number on top.

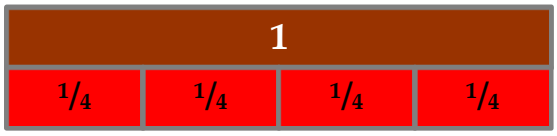
Possible questions:

- Show 30 ÷ 5 using a bar model
- What other number fact could we write using the same model (6 x 5 = 30)
- Taren made bags for the school fair. Each bag had 5 sweets in it, and she wanted to make 7 bags. How many sweets did she need altogether?
- Sami had 40 sweets. He shared them between himself and 9 friends. How many sweets did they get each?

Bar Models Progression - Year 2

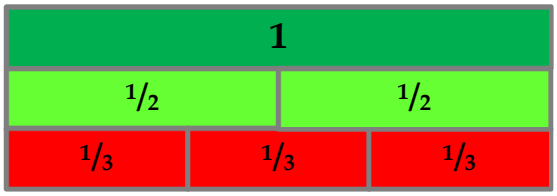
Fractions

- Recognise, find, name and write fractions 1/3 , 1/4 , 2/4 and 3/4 of a length, shape, set of objects or quantity
- Write simple fractions, for example 1/2 of 6 = 3 and recognise the equivalence of 2/4 and 1/2



Children continue to use the rods to represent different fractions: 1/3, 1/4, 2/4 and 3/4

What does the number on the bottom of the fraction tell us? (How many bars to use on the bottom of the model)



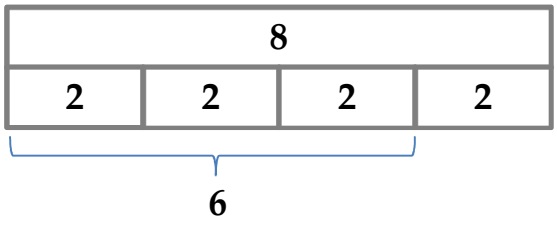
They begin to explore equivalences by matching up the bars.

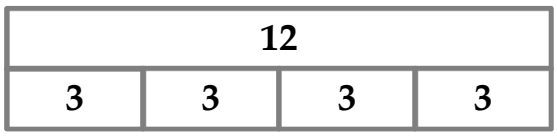
How many quarters are the same as a half?

Which is more, 1/3 or 1/2?

Possible questions:

- What is 3/4 of 8?
- True or false: 1/2 is more than 2/3
- A quarter of a number is 3.
  What is the number?
- Danny had 12 sweets. He gave 2/3 of them away. How many sweets did he have left?





Bar Models Progression - Year 3

Addition and Subtraction

- Add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction
- Estimate the answer to a calculation and use inverse operations to check answers
- Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

246	
78	166

78 + 166 = 246 166 + 78 = 246

246 – 78 = 166 246 – 166 = 78

413		
129	216	68

Children continue to link back to bar modelling to secure understanding when adding and subtracting larger numbers with column methods.

They continue to explore relationships within addition and subtraction to help when solving inverse operation problems, and to check answers to column addition and subtraction questions.

They also continue to use bar models to show addition of three numbers, and more complex subtraction.

Children begin to use bar models to represent more complex word problems. They begin with recapping simple, one-step problems before moving onto two-step problems, using their model to decide which operation to use.

Other concepts, such as money, should also be linked.

They should become comfortable with explaining how the model represents the problem, and should be able to think of their own word problems based on a given bar model.

£1.56		
35p	78p	44p

Write a word problem that this bar model could represent.

Can you write one that has 44p as the answer?

Possible questions:

- Show the bar model to represent 139 + 282
- Show the bar model to represent 319 – 148
- Work out the missing answer in this bar model. Write down 4 number sentences that are shown by the model
- Jen takes 38 pens out of a packet. There were 100 to start with. How many are left in the packet?
- Bill has £1.46 on one piggy bank, £2.39 in another and £1.27 in a third. How much money does he have altogether? Show this on a bar model.
- Katie wants a new game that costs £6.49. Her mum gives her £2.08 and her dad gives her £1.84. How much more does she need? Show this on a bar model.

£6.49		
£2.08	£1.84	?

£3.92

£2.08 + £1.84 = £3.92

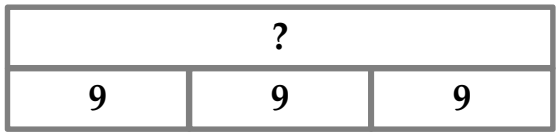
£6.49 - £3.92 = £2.57

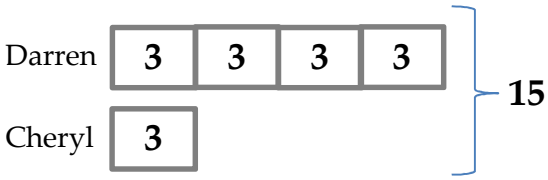
Bar Models Progression - Year 3

Multiplication and Division

- Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems

Children explore how bar models can help them to solve missing number problems in multiplication and division.
Some sweets are shared between 3 children. They get 9 each. How many sweets were there? (___ ÷ 3 = 9)





Children begin to use bar models in problems involving comparisons between two amounts.
Darren has 4 times as many cakes as Cheryl. Altogether, they have 15 cakes. How many do they each have?

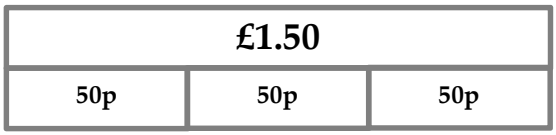
Possible questions:

- Five sea-lions share some fish at feeding time. Each one gets 7 fish. How many fish were shared out? Draw a bar model and write a number sentence to represent this problem.
- Some children are each given 8 pencils. Altogether, 32 pencils are given out. How many children were there? Draw a bar model and write a number sentence to represent this problem.
- A book case holds 17 books on each shelf. There are 6 shelves. How many books are there altogether?
- Lily has four times as many marbles as Charlie. He has 9 marbles. How many marbles do they have altogether?
- Rose has a third of the books that Thomas has. Altogether, they have 24 books. How many does Thomas have?

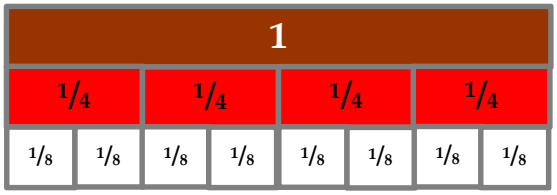
Bar Models Progression - Year 3

Fractions

- Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators
- Recognise and show, using diagrams, equivalent fractions with small denominators
- Add and subtract fractions with the same denominator within one whole
- Compare and order unit fractions, and fractions with the same denominators



Children begin to solve more complex 'fraction of a number' problems. *Eg. Poppy gets £1.50 pocket money. She shares the money with her sisters. Each sister gets 1/3. How much do they get?*

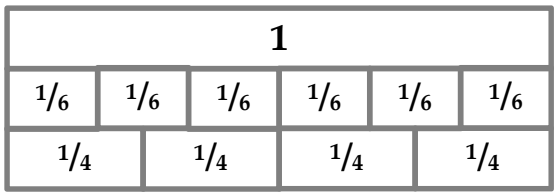


They use bar models to prove equivalences.

3/4 = 6/8

Using concrete resources, they show why ¼ + ¼ = 2/4 and not 2/8 (common error)

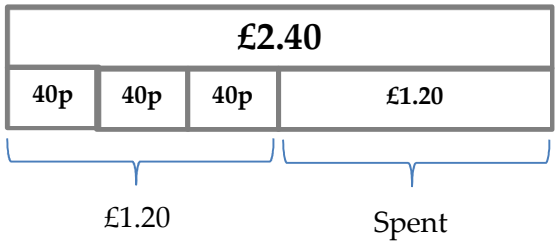
They compare different unit fractions on bar models, allowing them to make generalisations: *The bigger the denominator, the smaller the fraction.*



1/4 is bigger than 1/6

Possible questions:

- Emma has £3.60. She spends 1/3 of it at the shops. How much money does she have left?
- Which is more: 3/4 of 28 or 2/5 of 55?
- Denise spends 3/4 of the money in her piggy bank. She has £1.22 left. How much did she start with?
- Use a bar model to prove that 3/5 = 6/10
- Explain, using Cuisenaire rods, why 2/4 + 1/4 = 3/4
- Use bar models to explain how to order unit fractions
- Mary gets £2.40 pocket money. She spends half on a comic, and then shares the rest between herself and her 2 sisters. How much do her sisters get?



Bar Models Progression - Year 4

Addition and Subtraction

- Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- Estimate and use inverse operations to check answers to a calculation
- Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

5624	
1967	3657

1967 + 3657 = 5624 3657 + 1967 = 5624

5624 – 1967 = 3657 5624 – 3657 = 1967

1578		
613	729	236

1578 – 613 – 729 = 236

Children continue to link back to bar modelling to secure understanding when adding and subtracting larger numbers with column methods.

They continue to explore relationships within addition and subtraction to help when solving inverse operation problems, and to check answers to column addition and subtraction questions.

They also continue to use bar models to show addition of three numbers, and more complex subtraction.

Children begin to use bar models to represent more complex two-step word problems.

Other concepts, such as money, should also continue to be linked, and children should continue to think of their own problems to fit a given bar model.

A postman needs to deliver 2370 letters. He delivers 618 on Monday, 863 on Tuesday and 809 on Wednesday. How many does he have left to deliver?

2370			
618	863	809	?

The bar model should help children to see that they should add 618, 863 and 809. Once they have done this, the familiarity of the model will help them to see that they should take away the total from 2370.

Possible questions:

- Show the bar model to represent 2787 + 2009 + 1829
- Show the bar model to represent 7262 – 2786
- Work out the missing answer in this bar model. Write down 4 number sentences that are shown by the model
- There are 2131 books in the library. Year 2 borrow 117 books and Year 3 borrow 89 books. How many books will be left when Year 4 arrive in the library?
- There are 3711 people sat in a concert hall. 767 people have already left the concert. The hall can seat 5000 people altogether. How many seats were empty for the concert?

5000		
3711	767	?

4478

3711 + 767 = 4478

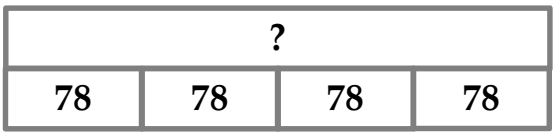
5000 – 4478 = 522

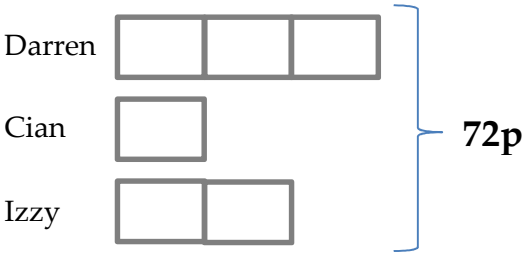
Bar Models Progression - Year 4

Multiplication and Division

- Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Children continue to explore how bar models can help them to solve missing number problems in multiplication and division.
4 children each have 78 Pokémon cards. How many do they have altogether?





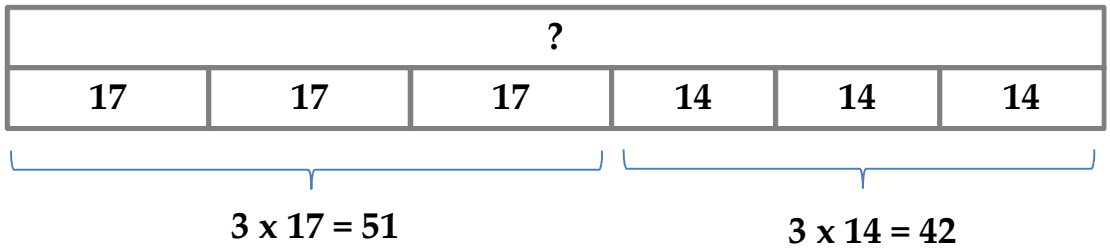
Children become more comfortable in using a 'stacked' bar model when using comparative information in scaling problems.
Darren has three times as much money as Cian, who has half the amount that Izzy has. Altogether, they have 72p. How much does each child have?

72 ÷ 6 = 12 so each box represents 12p.

Darren has 3 x 12 = 36p Cian has 12p Izzy has 2 x 12p = 24p

Possible questions:

- 5 children each win 16 points for their house. How many points do they win altogether?
- A minibus is full for 4 journeys during one day. Altogether, 36 people travel in the minibus that day. How many people does the minibus hold?
- 3 children each score 17 points in a game. Another 3 children each win 14 points. How many points do they win altogether?

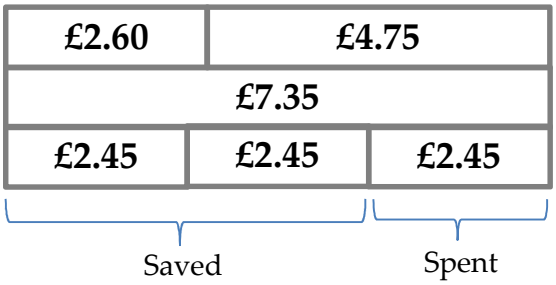


- Cara has twice as many marbles as Jen. Jen has a quarter of the amount that Lily has got. Altogether, they have 63 marbles. How many do they each have?

Bar Models Progression - Year 4

Fractions

- Recognise and show, using diagrams, families of common equivalent fractions
- Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- Add and subtract fractions with the same denominator



£2.60 + £4.75 = £7.35

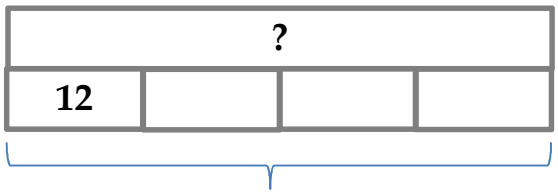
£7.35 ÷ 3 = £2.45

Children begin to solve more complex 'fraction of a number' problems. *Eg. Mary gets £2.60 pocket money. Her gran gives her another £4.75. She puts two thirds of the money in her piggy-bank and spends the rest. How much does she save?*

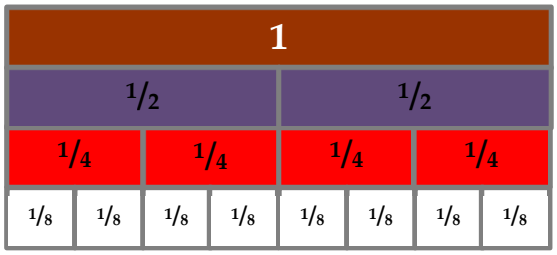
Children may start to adapt models. In this example, the 'whole' is placed at the bottom to give a clearer model.

Children can also begin to answer more complex questions about fractions of a number:

1/4 of a number is 12. What is the number?



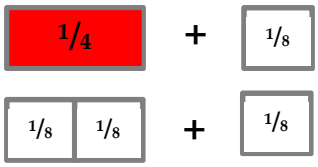
4 x 12 = 48



They continue to use bar models to prove equivalences, finding families of equivalent fractions.

1/2 = 2/4 = 4/8

Using concrete resources, remind children why ¼ + ¼ = 2/4 and not 2/8 (common error). Higher attaining pupils might begin to use bars to convert fractions in addition.



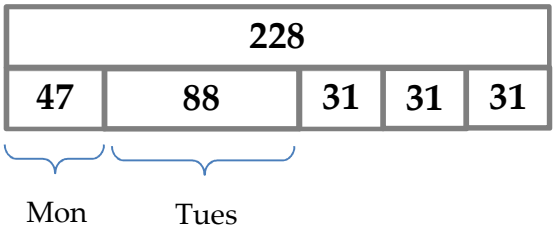
Possible questions:

- A pair of jeans costs 3/4 of their normal price in a sale. If they usually cost £28, how much do they cost in the sale?
- Use bar models to show why 1/2 + 1/4 = ¾
- Lauren has 1/3 of her pocket money left. She has £3.12, How much pocket money does Lauren get?

Bar Models Progression - Year 5

Addition, Subtraction, Multiplication and Division

- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

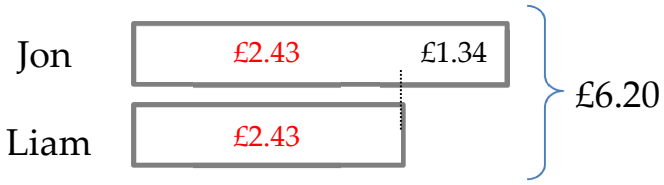


Children begin to combine the 4 operations more within multi-step word problem, using bar models to structure their thoughts and decide on appropriate calculations.

On Monday, Gita reads 47 pages of her book. She reads 88 pages the next day. If the book has 228 pages, and she splits the remaining pages between the next 3 days, how many pages does she read on these days?

They build of their knowledge of the stacked bar model, using it for other kinds of comparisons within questions.

Eg. Jon has £1.34 more than Liam. Altogether, they have £6.20. How much do they each have?

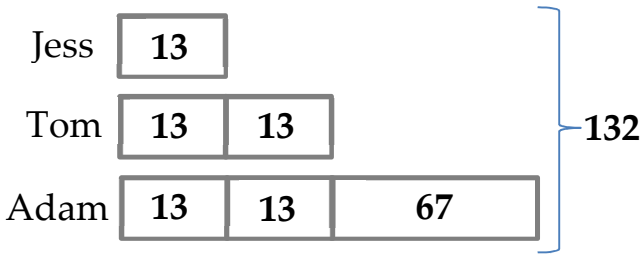


£6.20 – £1.34 = £4.86

£4.86 ÷ 2 = £2.43

Possible questions:

- Sam has half the amount of money Lara has. Emma has twice as much as Lara. If they have £1.61 altogether, how much do they each have?
- Tom has twice as many colouring pencils as Jess, but 67 less than Adam. They have 132 altogether. How many do they each have?
- Lara delivers a total of 567 letters. She delivered twice as many letters on Tuesday as she did on Monday. On Wednesday, she delivered 32 more than on Tuesday. How many did she deliver each day?
- Harry had £137 in his money box. He spent £65 on some computer games, and then shared what was left between himself and his 2 brothers. How much did each brother get?



132 – 67 = 65

65 ÷ 5 = 13

Jess: 13
Tom: 26
Adam: 93

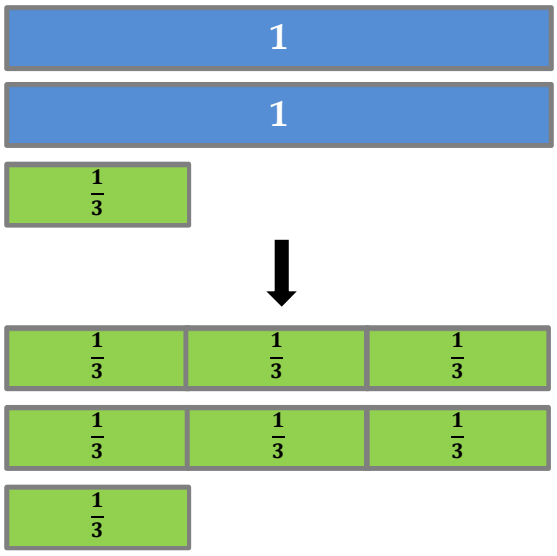
Bar Models Progression - Year 5

Fractions

- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number
- Add and subtract fractions with the same denominator and denominators that are multiples of the same number
- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

Children represent mixed numbers using the rods

Eg. $2\frac{1}{3}$

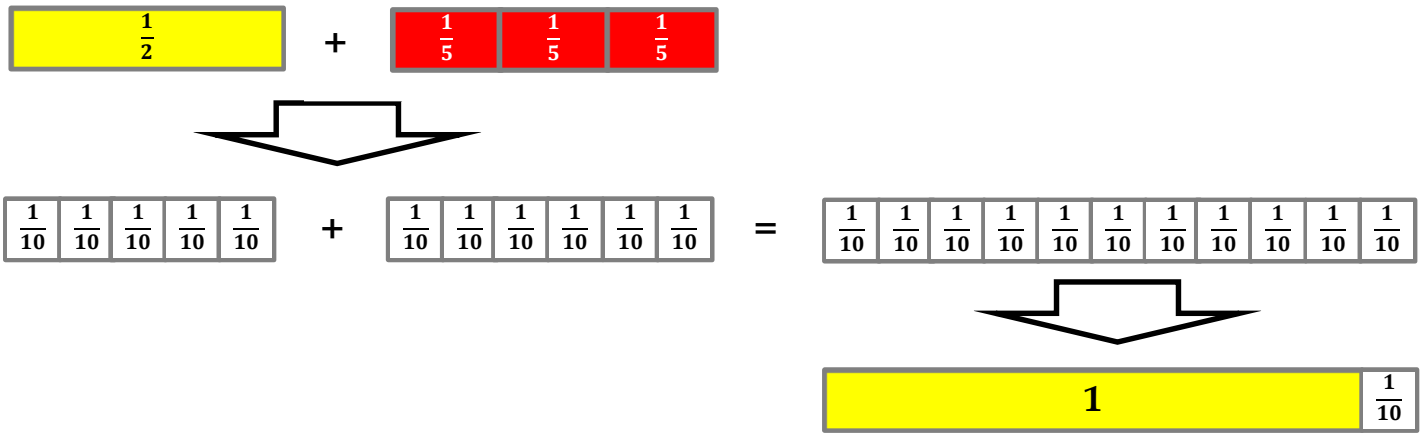


By substituting in 3 thirds for each 1 whole, children see that $2\frac{1}{3} = \frac{7}{3}$

Repeat the activity but start with an improper fraction. Eg. Show that $\frac{11}{4} = 2\frac{3}{4}$

Add and subtract fractions with different denominators:

$$\frac{1}{2} + \frac{3}{5} = \frac{5}{10} + \frac{6}{10} = \frac{11}{10} = 1\frac{1}{10}$$



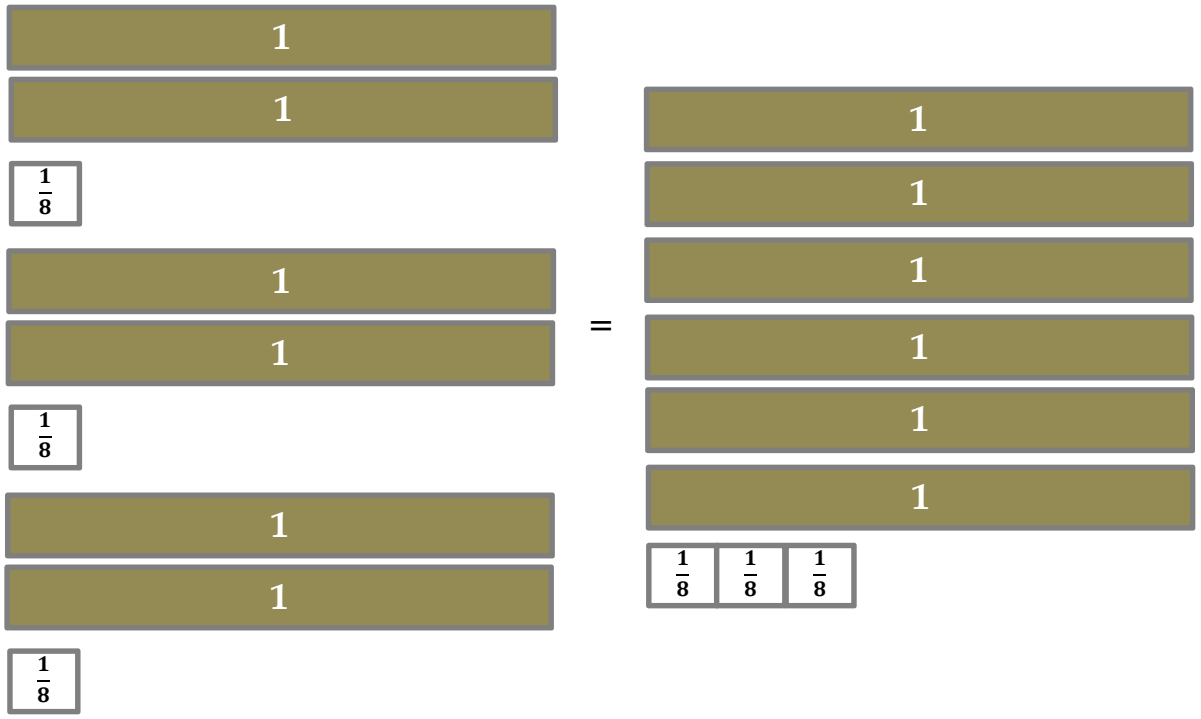
Primary Solutions

Bar Models Progression - Year 5

Multiply proper fractions:

$3 \times \frac{1}{5}$

$3 \times 2\frac{1}{8}$



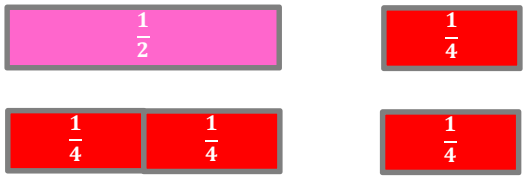
Possible questions:

Change $\frac{14}{6}$ into a mixed number.

Change $3\frac{2}{3}$ into an improper fraction

Daisy ate one and a half bread rolls every day for 5 days. How many rolls did she eat altogether?

Luke ate $\frac{1}{4}$ of a pizza one day and $\frac{1}{2}$ of it the next. How much of the pizza did he eat altogether?



$$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

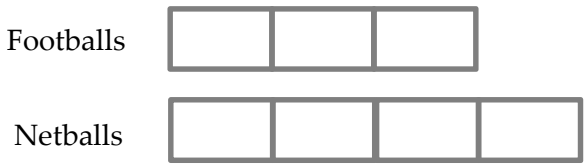
Bar Models Progression - Year 6

Ratio and Proportion

- solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
- solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison

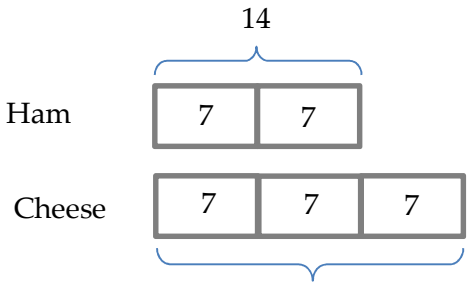
Children express ratios using bar models.

For example: There are 3 footballs for every 4 netballs in a bag.



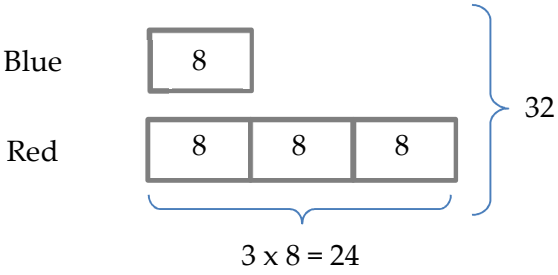
They begin to solve problems such as:

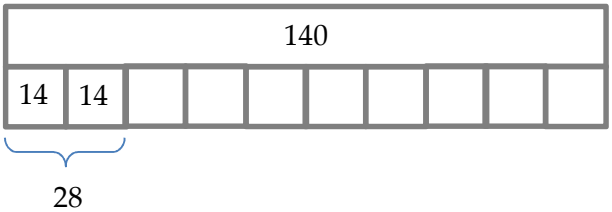
There are 2 ham sandwiches for every 3 cheese sandwiches in a shop. If there are 14 ham sandwiches, how many cheese are there?



Variations of similar questions can then be explored:

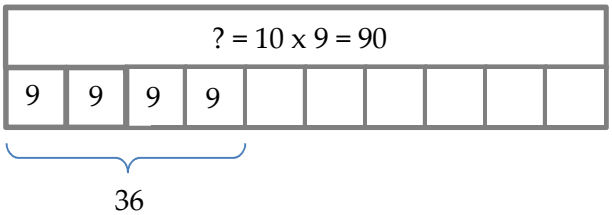
There are 3 red balls for every blue ball in a bag. If there are 32 balls in the bag altogether, how many of each colour are there?





Children understand that bar models can be used to work out some percentages of numbers:

Find 20% of 140



Children explore inverse percentages. For example:

40% of a number is 36. What is the number?

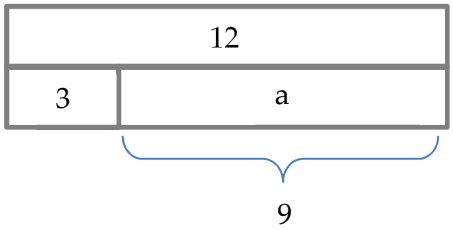
Bar Models Progression - Year 6

Algebra

- express missing number problems algebraically
- find pairs of numbers that satisfy an equation with two unknowns

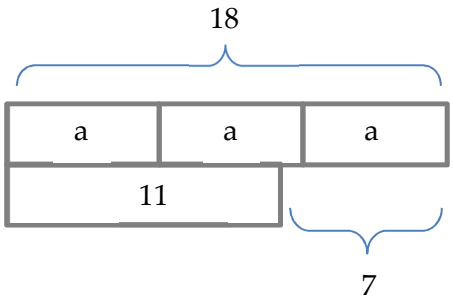
Children begin to explore algebra by representing missing numbers as letters in a bar model.

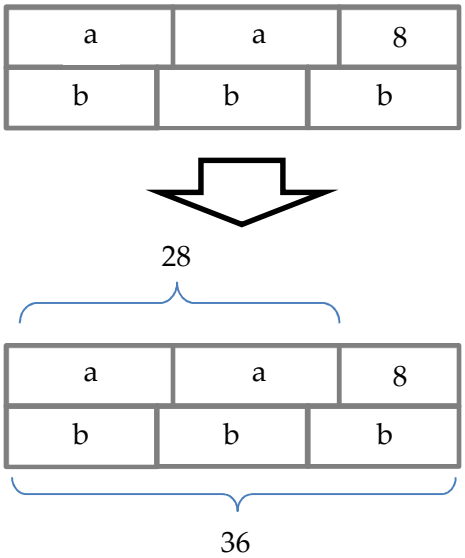
For example: 3 + a = 12



Questions become more complex, with multiples of a missing value involved.

For example: 3a – 7 = 11





Children develop this model to take account of 2 variables.

For example: 2a + 8 = 3b.

If a = 14, what is b?

Possible questions:

- If t – 4 = 15, what is t?
- If 6 + 4s = 58, what is s?
- If 2x + 7 = y: What is y if x = 15? What is x if y = 33?